it has been changed to --di--. Also the figure did not show the output portion of the fiber which has now been added to show the output --do--.

The objections to the Specification have been cured as follows:

RELATED APPLICATIONS has been changed to define this application as a divisional. Changes to the figure descriptions have been made with respect to Figs. 2, 4, 6, and 7. The objection on page 4 relating to "(di)" and "(do)" has been cured by changing Fig. 1 to indicate di as an input and do as an output. The objection to page 4, line 14 has been cured by defining "N" and "η_i". These are known reference terms. N is a common mathematical designation for the number of segments used for the calculation. It represents the number of positions to which the calculation is applied. η_i is the fraction of the power carried by core guided modes. This is shown in the attached pages from Optical Waveguide Theory (Exhibit A). These definitions have been added to the specification.

On page 7, line 17 the word "charges" has been changed to -- changes --.

The applicant responds to the claim objections as follows:

With respect to the objection to claim 6, the objection is cured by the amendment to the claim 1 such that it recites "said fiber having at least one parameter…", rather than the sheath. This amendment corrects a typographical error. Claim 7 has been amended to recite that the sheath includes a cladding. Claim 8 appears to be correct, the objection is traversed. Claim 27 has been amended to recite an input end.

The following remarks relate to the claim rejections.

Claims 1 and 17 were rejected under section 102. The Examiner explained the rejection as follows.

Claims 1 and 17 are rejected under 35 U.S.C. 102(b) as being anticipated by Tarbox.

With regard to Claim 1 and 17, Tarbox discloses an optical fiber, said optical fiber (See 18 in Figures 1 or 2) having a core and a sheath (See col. 1, lines 8-11), said sheath having at least one parameter (See 18 in Figures 1 or 2; col. 2, line 66-col.3, line 9 for description of coiling of fiber to reduce fiber stressing that can result in variations of fiber attenuation along the length of the fiber) that varies from an input end of said fiber to an output end thereof in a manner to maintain a constant power loss per unit length over the length of said fiber (col. 2, line 66-col. 3, line 9).

This rejection is traversed.

The present invention solves a problem relating to the existence of spatial transient effect of light traveling in multi-mode fibers. This is particularly brought out in the comment in the specification at page 4 which states "it is clear that the power loss per unit length at each position along a distributed optical fiber sensor is different from the power loss per unit length at any other position along the fiber".

Claim 1 recites a fiber that has at least one parameter that varies from an input end to an output end in a manner to maintain a constant power loss per unit length, over the length from the input end to the output end.

Referring to the Tarbox reference it is noted that Tarbox makes a reference to optical fiber attenuators and speaks to the attenuation per unit length of an optical fiber. Notably, Tarbox states "preferably the optical fiber of the device has as a substantially uniform attenuation to through its length since if this is known the optical fiber may be cut to a predetermined length to provide a required attenuation value" (col. 2, lines 3-6)

and "for example in the embodiments of lengths 18 of optical fiber of the devices are cut from fibers having a known attenuation per unit length (which is substantially uniform throughout the length) to predetermined lengths to provide required attenuation values" (col. 2, lines 57-65). In particular, Tarbox does not vary a parameter along the length of the fiber nor does he even mention varying a parameter from one end of the fiber to another.

By contrast, for sensor applications, light is introduced at the front of the fiber and light is lost faster in the spatial transient region than it is further along the fiber. The goal is to make a fiber that has a lower intrinsic or inherent loss in the spatial transient region.

The Examiner has cited various recitations in Tarbox as showing such a parameter. But it is submitted that the portions of Tarbox cited do not teach such a parameter. Portions cited by the Examiner are cited as showing "description of coiling of fiber to reduce fiber stressing that can result in variations of fiber attenuation along the length of the fiber." As so described by the Examiner, this does not describe a parameter of a fiber that varies so as to maintain a constant power loss per unit length of the fiber.

Referring to the specific portion of Tarbox cited by the Examiner: col. 2 from line 66 to col. 3, line 9. This teaches coiling a fiber in a stress free manner so that its attenuation is not altered. For example the diameter of the turns of the coiled length is great enough not to cause mechanical stressing of the fiber.

This is unrelated to having a parameter that varies from one end to the other end to maintain a constant power loss.

In fact nowhere does Tarbox refer to the end result of having a constant power

loss. Tarbox does not teach varying a parameter along the length of the fiber. In fact Tarbox does not appreciate the phenomenon of spatial transients in an optical fiber. Consequently, where spatial transients are in effect, the Tarbox construction would not work because Tarbox wants his device to have substantially uniform attenuation through the length of the fiber, that is the fiber has a known attenuation per unit length (which is substantially uniform throughout the length). Col. 2, line 3-6 and col. 2, line 60-65.

The present invention recognizes that optical fibers are characterized by spatial transients for transmitted light which causes the sensitivity of chemically or physically sensitive fibers to vary from point to point. This is particularly true of multi-mode fibers with lossy (e.g. absorber-doped) coatings where light does not reach equilibrium for a considerable distance. Such a varying response is due to a spatial transient, associated with the existence of radiation modes, and a stronger attenuation of higher order bound modes. Accordingly, sensor response over the length of the fiber is not constant.

To solve the problem presented, it has been discovered according to the present invention to compensate for the spatial transient in distributed fiber optic sensors, to have a constant sensitivity over the length of a distributed fiber, by introducing a change in the light-guiding characteristics of the fiber to compensate for the effects of spatial transients.

A number of parameters associated with fibers for particular sensor modes are disclosed in this application. These are broadly referred to as parameters, and in the use of Claim 1, the parameter varies over the length in a manner to maintain constant power loss per unit length.

In Claim 17 the cladding is sensitive to a physical quantity and the fiber has a parameter that varies over the length to make the power loss vary in a controlled way.

An example of the invention is shown with reference to Fig. 2 that shows the charge in radiated power per unit length over the length. Recognizing this, the invention is that a variety of parameters associated with the fiber construction can be varied in order to compensate.

Fig 4 and the accompanying text show one embodiment of the invention with reference to the core/cladding refractive index ration.

The invention refers to a problem in a length of fiber used as a sensor, where an event occurs at a discrete part of area along the length. In the invention the fiber is structured to exhibit a length-invariant response to the event.

Claims 3, 5, 21, 26-27 were rejected under section 103 as being unpatentable over Hamburger et al in view of Tarbox. This rejection is traversed.

The comments above with respect to the invention as compared to the Tarbox teaching are adopted with respect to this rejection. Inasmuch as Tarbox does not teach an optical fiber having at least one parameter that varies from input end to an output end thereof in a manner to maintain a constant power loss per unit length over the length of the fiber the proposed combination would not render the referenced claims obvious.

Claims 2 and 4 were rejected under section 103 as being unpatentable over Hamburger et al in view of Tarbox as applied to claim 1 and further in view of Cramp et al. Again, inasmuch as Tarbox does not disclose the invention as recited in claim 1 the combination proposed by the Examiner would not render the claims obvious.

A one month extension of time is requested. A check for $55.00 is enclosed to pay for the extension of time. If any further extension of time is required for this filing such extension is hereby requested and if any additional fee is required for this filing it may be charged to deposit account no. 50-1054.

Respectfully submitted,

L. S. Cohen

Lawrence S. Cohen, Esq.
Date: November 25, 2002
Reg. No. 25,225

Law Offices of Lawrence S. Cohen
10960 Wilshire Boulevard, Suite #830
Los Angeles, CA 90024
310-231-6898;310-231-6899 Fax

MARKED UP VERSION OF SPECIFICATION CHANGES

RELATED APPLICATIONS

This is a [continuation] divisional of patent application Serial No. 09/334,845, filed on 06/16/99 and assigned to the assignee of the present application, the entire content of which is incorporated herein by reference.

BRIEF DESCRIPTION OF THE DRAWING

Fig['s]. 2 is a graph of power transmitted vs. fiber position for prior art apparatus;

Fig. 4 is a graph of refractive index variation with distance from the launch end of a fiber in accordance with the principles of this invention.[.]

Fig. 6 is a block diagram of a system using the fiber of Fig[']s. 1 and 3.

Fig. 7 is a block diagram of a further system using the fiber of Fig[']s. 1 and 3.

DETAILED DESCRIPTION OF ILLUSTRATIVE EMBODIMENTS OF THE INVENTION

where $\mathbf{P_o}$ is the power at the input end of the fiber, $\mathbf{P}$ is the power at a distance 1 from the input end, $\mathbf{a}$ is the core radius, $\boldsymbol{\alpha}$ is the absorption coefficient of the cladding, [and] $\mathbf{n_{co}}$ and $\mathbf{n_{cla}}$ are the core and cladding refractive indices[.], **N** is a common mathematical designation for the number of segments used for the calculation; it represents the number of positions to which the calculation is applied, and $\boldsymbol{\eta_i}$ is the fraction of the power carried by core guided modes.

An optical fiber having a core of F-2 Schott glass with a diameter of 100 micrometers and a cladding of polymer with a thickness of 20 micrometers was fabricated and a twenty meter length of the fiber was tested at 850nm (active wavelength) as a moisture sensor. A reference wavelength of 1300 nm also was used. A dry reference measurement was made and then increasing lengths of 10 cm, 20 cm, and 50 cm were wetted. A final reading was taken after allowing the 50 cm wet length to air dry for one hour. At 1300 nm only a slight change (< 1 dB) in output intensity occurs after an input signal propagates more than 10 meters along the fiber. A significant change of 2.4 dB is detected at 850 nm. The OTDR was able to determine the location of the moisture site to within 1 cm using 850 nm light and the 1300 nm light was capable of being used compensate for transmission [charges] changes due to effects other than moisture.

MARKED UP VERSION OF THE CLAIMS

Claim 1 (first amendment). An optical fiber, said fiber having a core and a sheath, said [sheath] <u>fiber</u> having at least one parameter that varies from an input end of said fiber to an output end thereof in a manner to maintain a constant power loss per unit length over the length of said fiber.

Claim 27 (first amendment). An optical fiber as in claim 26 said fiber having a light source at [the] <u>an</u> input end thereof.[.]

Claim 7 (first amendment). An optical fiber as in claim 1 wherein said <u>sheath includes a cladding and said</u> one parameter comprises the core/cladding refractive index ratio.

Optical Waveguide Theory

ALLAN W. SNYDER

JOHN D. LOVE

Institute of Advanced Studies
Australian National University
Canberra, Australia

CHAPMAN & HALL MEDICAL
London · Weinheim · New York · Tokyo · Melbourne · Madras

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EXHIBIT A
RESPONSE- 25 NOV 02

jth forward- and backward-propagating modes, we find from Eq. (11–9) that

$$\boxed{\begin{aligned} P_j &= \tfrac{1}{2}|a_j|^2 \int_{A_\infty} \mathbf{e}_j \times \mathbf{h}_j^* \cdot \hat{\mathbf{z}}\, dA, \qquad (11\text{–}21a) \\ P_{-j} &= -\tfrac{1}{2}|a_j|^2 \int_{A_\infty} \mathbf{e}_j \times \mathbf{h}_j^* \cdot \hat{\mathbf{z}}\, dA. \qquad (11\text{–}21b) \end{aligned}}$$

Thus $P_j > 0$ represents power propagating in the positive z-direction in Fig. 11–1(a), and $P_{-j} < 0$ represents power propagating in the negative z-direction. In terms of the mode normalization of Eq. (11–12)

$$\boxed{P_j = |a_j|^2 N_j; \qquad P_{-j} = -|a_{-j}|^2 N_j,} \qquad (11\text{–}22a)$$

while for the orthonormal modes of Eq. (11–16)

$$\boxed{P_j = |a_j|^2; \qquad P_{-j} = -|a_{-j}|^2.} \qquad (11\text{–}22b)$$

11–8 Fraction of modal power in the core

The characteristically different forms of the refractive index in the core and cladding in Fig. 11–1(b) lead to distinctly different behavior of the waveguide fields in the two regions. However, we can often gain insight into the relative behavior of the fields in the two regions by examining the fraction of a mode's power that propagates within the core. We define a parameter η_j as

$$\eta_j = \frac{\text{power flow within the core}}{\text{total power flow of the mode}}. \qquad (11\text{–}23)$$

For a nonabsorbing waveguide, we deduce from Eq. (11–21) that

$$\boxed{\eta_j = \eta_{-j} = \frac{\dfrac{1}{2}\displaystyle\int_{A_{co}} \mathbf{e}_j \times \mathbf{h}_j^* \cdot \hat{\mathbf{z}}\, dA}{\dfrac{1}{2}\displaystyle\int_{A_\infty} \mathbf{e}_j \times \mathbf{h}_j^* \cdot \hat{\mathbf{z}}\, dA},} \qquad (11\text{–}24)$$

where A_{co} is the core cross-section, and A_∞ is the infinite cross-section. This expression is also useful for describing slightly absorbing waveguides, discussed later in the chapter.

11–9 Total guided power

We define P_{tot} to be the *total, time-averaged power* propagating along an optical waveguide *in the increasing z-direction* of Fig. 11–1(a). In terms of the

total fields of Eq. (11–2), P_{tot} i
infinite cross-section A_∞

P_{tot} =

We can divide P_{tot} into the po
P_{bd}, and the power of the ra

If we substitute Eqs. (11–2) a
nality condition, Eq. (11–13),
Eq. (11–21), we find from Eq

$$P_{bd} = \sum_{j=1}^{M} \{P_j + \ldots$$

where a_j and a_{-j} are the a
propagating modes, and N_j is
guided power propagating in
all forward-propagating mod
modes. At any position along
the difference between P_{tot} a

11–10 Fraction of total pow

The total power P_{tot} of Eq. (1
and the total power in the cl

P_{co} =

In regions sufficiently far alo
the radiation field within the
the fields within the core is giv
However, P_{co} is still a complic
orthogonal over any finite cr
which only the forward-prop
waveguide. Substituting Eq. (
gives

$$P_{co} = \frac{1}{2}\sum_j \sum_k a_j a_k^* \ldots$$

where $j, k > 0$. Rearranging ar

total fields of Eq. (11–2), P_{tot} is given by the integrated Poynting vector over the infinite cross-section A_∞

$$P_{tot} = \frac{1}{2}\mathrm{Re}\int_{A_\infty} \mathbb{E}\times\mathbb{H}^*\cdot\hat{\mathbf{z}}\,dA. \tag{11–25}$$

We can divide P_{tot} into the power of the guided, or bound, portion of the fields, P_{bd}, and the power of the radiation field, P_{rad}, so that

$$P_{tot} = P_{bd} + P_{rad}. \tag{11–26}$$

If we substitute Eqs. (11–2) and (11–3) into Eq. (11–25), apply the orthogonality condition, Eq. (11–13), and recall the definition of modal power from Eq. (11–21), we find from Eq. (11–26) that for a nonabsorbing waveguide

$$\boxed{P_{bd} = \sum_{j=1}^{M}\{P_j + P_{-j}\} = \sum_{j=1}^{M}|a_j|^2 N_j - \sum_{j=1}^{M}|a_{-j}|^2 N_j,} \tag{11–27}$$

where a_j and a_{-j} are the amplitudes of the jth forward- and backward-propagating modes, and N_j is the normalization of Eq. (11–12). Thus, the total guided power propagating in the positive z-direction is equal to the power in all forward-propagating modes minus the power in all backward-propagating modes. At any position along the waveguide, the power in the radiation field is the difference between P_{tot} and P_{bd}.

11–10 Fraction of total power in the core

The total power P_{tot} of Eq. (11–25) is the sum of the *total power in the core*, P_{co}, and the total power in the cladding, where

$$P_{co} = \frac{1}{2}\mathrm{Re}\left\{\int_{A_{co}} \mathbb{E}\times\mathbb{H}^*\cdot\hat{\mathbf{z}}\,dA\right\}. \tag{11–28}$$

In regions sufficiently far along the waveguide from any source of excitation, the radiation field within the core becomes negligible, and the total power of the fields within the core is given accurately by the bound modes in Eq. (11–2). However, P_{co} is still a complicated expression because *the bound modes are not orthogonal over any finite cross-section.* To see this, consider a situation in which only the forward-propagating modes are excited on a nonabsorbing waveguide. Substituting Eq. (11–2) into Eq. (11–28) and applyiing Eq. (11–3) gives

$$P_{co} = \frac{1}{2}\sum_j\sum_k a_j a_k^* \exp\{i(\beta_j - \beta_k)z\}\int_{A_{co}} \mathbf{e}_j\times\mathbf{h}_k^*\cdot\hat{\mathbf{z}}\,dA, \tag{11–29}$$

where $j, k > 0$. Rearranging and employing the definitions of Eqs. (11–21a) and

Table 13–2 Properties of bound modes on weakly guiding waveguides. Parameters are defined inside the back cover. The modal amplitude a depends on the source of illumination, and A_{co} is the core cross-section. We assume $\mathbf{e}_t$, Ψ and F_t are real on nonabsorbing

		Arbitrary waveguide
Refractive-index profile		$n_{co}^2\{1-2\Delta f(x,y)\}$
Orthogonality $j \neq k$		$\int_{A_\infty} \mathbf{e}_{tj}\cdot\mathbf{e}_{tk}\,\mathrm{d}A = 0$
Normalization	$\tilde{N}$	$\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_{A_\infty} \mathbf{e}_t^2\,\mathrm{d}A$
Intensity or power density	$\tilde{S}$	$\lvert a\rvert^2\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\mathbf{e}_t^2$
Modal power	$\tilde{P}$	$\lvert a\rvert^2\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_{A_\infty} \mathbf{e}_t^2\,\mathrm{d}A$
Fraction of power in the core	$\tilde{\eta}$	$\int_{A_{co}} \mathbf{e}_t^2\,\mathrm{d}A \Big/ \int_{A_\infty} \mathbf{e}_t^2\,\mathrm{d}A$
Power attenuation coefficient	$\tilde{\gamma}$	$2k\int_{A_\infty} n^i\,\mathbf{e}_t^2\,\mathrm{d}A \Big/ \int_{A_\infty} \mathbf{e}_t^2\,\mathrm{d}A$
Propagation constant	$\tilde{\beta}^2$	$\dfrac{\int_{A_\infty}\{(kn\mathbf{e}_t)^2-(\nabla_t\cdot\mathbf{e}_t)^2\}\,\mathrm{d}A}{\int_{A_\infty}\mathbf{e}_t^2\,\mathrm{d}A}$
Modal parameter	$\tilde{U}^2$	$\dfrac{\int_{A_\infty}\{fV^2\mathbf{e}_t^2+(\rho\nabla_t\cdot\mathbf{e}_t)^2\}\,\mathrm{d}A}{\int_{A_\infty}\mathbf{e}_t^2\,\mathrm{d}A}$
Useful derivative	$\frac{\mathrm{d}\tilde{U}}{\mathrm{d}V}$	$\frac{\tilde{V}}{\tilde{U}}\int_{A_\infty} f\mathbf{e}_t^2\,\mathrm{d}A \Big/ \int_{A_\infty} \mathbf{e}_t^2\,\mathrm{d}A$
Group velocity	$\tilde{v}_g$	$\frac{c\tilde{\beta}}{k}\int_{A_\infty} \mathbf{e}_t^2\,\mathrm{d}A \Big/ \int_{A_\infty} n^2\mathbf{e}_t^2\,\mathrm{d}A$
Distortion parameter	$\tilde{D}$	$-\frac{\mathrm{d}^2}{\mathrm{d}V^2}\left(\frac{\tilde{U}^2}{2V}\right)$

waveguides, and are approx
The equations satisfied by Ψ
solutions of the scalar wave
defined in Table 30–1, page

Noncircular waveguide
$n_{co}^2\{1-2\Delta f(x,y)\}$
$\int_{A_\infty} \Psi_j\Psi_k\,\mathrm{d}A = 0$
$\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_{A_\infty} \Psi^2\,\mathrm{d}A$
$\lvert a\rvert^2\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\Psi^2$
$\lvert a\rvert^2\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_{A_\infty} \Psi^2\,\mathrm{d}A$
$\int_{A_{co}} \Psi^2\,\mathrm{d}A \Big/ \int_{A_\infty} \Psi^2\,\mathrm{d}A$
$2k\int_{A_\infty} n^i\Psi^2\,\mathrm{d}A \Big/ \int_{A_\infty} \Psi^2\,\mathrm{d}A$
$\dfrac{\int_{A_\infty}\{(kn\Psi)^2-(\nabla_t\Psi)^2\}\,\mathrm{d}A}{\int_{A_\infty}\Psi^2\,\mathrm{d}A}$
$\dfrac{\int_{A_\infty}\{fV^2\Psi^2+(\rho\nabla_t\Psi)^2\}\,\mathrm{d}A}{\int_{A_\infty}\Psi^2\,\mathrm{d}A}$
$\frac{V}{\tilde{U}}\int_{A_\infty} f\Psi^2\,\mathrm{d}A \Big/ \int_{A_\infty} \Psi^2\,\mathrm{d}A$
$\frac{c\tilde{\beta}}{k}\int_{A_\infty} \Psi^2\,\mathrm{d}A \Big/ \int_{A_\infty} n^2\Psi^2\,\mathrm{d}$
$-\frac{\mathrm{d}^2}{\mathrm{d}V^2}\left(\frac{\tilde{U}^2}{2V}\right)$

/eakly guiding wave- :k cover. The modal)n, and A_{co} is the core eal on nonabsorbing

vaveguide
$f(x, y)\}$
$dA = 0$
$^2\int_{A_\infty} e_t^2 dA$
$\frac{0}{0})^{1/2} e_t^2$
$\frac{0}{0})^{1/2}\int_{A_\infty} e_t^2 dA$
$/\int_{A_\infty} e_t^2 dA$
$^2 dA/\int_{A_\infty} e_t^2 dA$
$^2-(\nabla_t\cdot e_t)^2\}dA$ / $e_t^2 dA$
$e_t^2+(\rho\nabla_t\cdot e_t)^2\}dA$ / $e_t^2 dA$
$A/\int_{A_\infty} e_t^2 dA$
$A/\int_{A_\infty} n^2 e_t^2 dA$
$\frac{2}{V})$

waveguides, and are approximately real on an absorbing waveguide if $n^i \ll n^r \cong n_{co}$. The equations satisfied by Ψ and F_l are in Table 13–1, and $F_l^{(j)}$, $F_l^{(k)}$ denote different solutions of the scalar wave equation for the same value of l. Vector operators are defined in Table 30–1, page 592.

Noncircular waveguide	*Circular fiber*
$n_{co}^2\{1-2\Delta f(x,y)\}$	$n_{co}^2\{1-2\Delta f(R)\}$
$\int_{A_\infty}\Psi_j\Psi_k\,dA=0$	$\int_0^\infty F_l^{(j)}F_l^{(k)}\,R\,dR=0$
$\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_{A_\infty}\Psi^2\,dA$	$\pi\rho^2 n_{co}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_0^\infty F_l^2\,R\,dR$
$\lvert a\rvert^2\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\Psi^2$	$\lvert a\rvert^2\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}F_l^2$
$\lvert a\rvert^2\frac{n_{co}}{2}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_{A_\infty}\Psi^2\,dA$	$\pi\rho^2\lvert a\rvert^2 n_{co}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\int_0^\infty F_l^2\,R\,dR$
$\int_{A_{co}}\Psi^2\,dA\Big/\int_{A_\infty}\Psi^2\,dA$	$\int_0^1 F_l^2\,R\,dR\Big/\int_0^\infty F_l^2\,R\,dR$
$2k\int_{A_\infty}n^i\Psi^2\,dA\Big/\int_{A_\infty}\Psi^2\,dA$	$2k\int_0^\infty n^i F_l^2\,R\,dR\Big/\int_0^\infty F_l^2\,R\,dR$
$\dfrac{\int_{A_\infty}\{(kn\Psi)^2-(\nabla_t\Psi)^2\}\,dA}{\int_{A_\infty}\Psi^2\,dA}$	$\dfrac{\int_0^\infty\left\{(k\rho nF_l)^2-\left(\frac{dF_l}{dR}\right)^2-\left(l\frac{F_l}{R}\right)^2\right\}R\,dR}{\rho^2\int_0^\infty F_l^2\,R\,dR}$
$\dfrac{\int_{A_\infty}\{fV^2\Psi^2+(\rho\nabla_t\Psi)^2\}\,dA}{\int_{A_\infty}\Psi^2\,dA}$	$\dfrac{\int_0^\infty\left\{\left(\frac{dF_l}{dR}\right)^2+\left(\frac{l^2}{R^2}+V^2 f\right)F_l^2\right\}R\,dR}{\int_0^\infty F_l^2\,R\,dR}$
$\frac{V}{\tilde{U}}\int_{A_\infty}f\Psi^2\,dA\Big/\int_{A_\infty}\Psi^2\,dA$	$\frac{V}{\tilde{U}}\int_0^\infty fF_l^2\,R\,dR\Big/\int_0^\infty F_l^2\,R\,dR$
$\frac{c\tilde{\beta}}{k}\int_{A_\infty}\Psi^2\,dA\Big/\int_{A_\infty}n^2\Psi^2\,dA$	$c\frac{\tilde{\beta}}{k}\int_0^\infty F_l^2\,R\,dR\Big/\int_0^\infty n^2F_l^2\,R\,dR$
$-\frac{d^2}{dV^2}\left(\frac{\tilde{U}^2}{2V}\right)$	$-\frac{d^2}{dV^2}\left(\frac{\tilde{U}^2}{2V}\right)$